

Essays in infinite dynamic games

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Valorisation

The purpose of this valorisation is to provide an answer to the following question: “What value can society derive from the research presented in this thesis?”

To answer this question we first need to realize that no research exists in a vacuum and neither does the research discussed in this thesis. This observation is especially important given the theoretical and abstract nature of this thesis. Because of this the immediate opportunities for concrete applications are limited. The findings of this thesis contribute to the existing academic literature on dynamic games. How precisely this research contributes to this existing academic literature was already answered elaborately in each of the chapters, there is therefore no need to repeat it here. Instead, I will attempt to sketch the value of the study of dynamic games.

The mathematical framework of dynamic games can be used to model interactions between players who encounter each other in different environments. These players often face a trade-off between choosing a good action now and trying to ensure a favourable future game. Such games are of interest to economists as they provide a natural framework to study the interactions of competing players over time. In Chapter 3 we studied a concrete dynamic game which had a clear economic motivation. The mathematical framework we developed there, provided insight in how decreasing competition can lead to expected outcome inequality, even when all competitors are identical and get equal opportunities. Apart from this, the economic applications of dynamic games are numerous. Other examples include the study of bargaining procedures, reputation building and the (over)exploitation of a common resource.

Dynamic games are also of interest to computer scientists. One application is the study of reactive systems. Such systems must be able to continuously react to uncontrollable events in the environment in which they interact. An example mentioned by Bruyère (2017) is the autopilot of a plane which controls the speed. This system must adjust the speed continuously depending on the weather conditions. This can be modelled by a two player zero-sum game. The two players in question are the system and the environment. The environment is assumed to be hostile and its objective is to do whatever it can to make the system fail. If the system can always succeed, no matter what the environment does, then the system has a winning strategy. Also non-zero sum dynamic games are of interest to computer science as they can be used to model complex systems, i.e. systems which consist of multiple components where each component has its own objective function.

In the applications of infinite dynamic games in computer science the payoff-function is often not “continuous at infinity”. Which implies that events that happen in a distant future can have a substantial impact on the outcome of the game. One example of such a payoff function is the reachability condition. Here the system gets a payoff of one if it

reaches a certain set of states and zero otherwise. The effect of the non-continuity of the payoff-functions on the complexity of the mathematical analysis of the game should not be underestimated.

To analyse an infinite horizon dynamic game where the payoff function is continuous at infinity, the game can essentially be analyzed as a finite horizon game, given that one is indifferent to small changes in the payoff. The analysis of finite horizon dynamic games can be easily done using the technique of backward induction. Unfortunately, this technique will not necessarily be successful in infinite games with discontinuous payoff-functions. Hence different techniques need to be developed for the analysis of such games.

It is in this context that Chapters 1 and 2 aim to contribute. In both chapters the discontinuity of the payoff functions plays a central role. Chapter 1 studies two player zero-sum stochastic games where players have universally measurable payoff functions. The class of universally measurable payoff functions is very broad and essentially includes all possible payoff functions for which an expected payoff can be computed for any given strategy profile. Chapter 2 defines a new type of continuity which we call individually upper-semi continuity, which extends the notation of upper semi-continuity. We showed, by means of construction, that a subgame perfect ϵ equilibrium always exists in dynamic games with almost perfect information in which all players have individually upper semicontinuous payoff functions.